Concours Putnam

Atelier de Pratique
Le jeudi, 14 novembre 12h30-13h30
La salle 5448 Pav. André Aisenstad

Jeus et invariants

1. Given an 8×8 grid representing a chessboard. At each step, any two columns or any two rows may be swapped. Is it possible to make the top half of the grid white and the bottom half black in a few steps?

2. Consider the following two-player game. Initially, there is a large number of tokens on the table. Each player in their turn can remove between 1 and 10 tokens. The player who must take the last token loses. Give a strategy for the first player to win if we start with 99 tokens; and a strategy for the second player to win if we start with 100 tokens.

3. The numbers $1, \frac{1}{2}, ..., \frac{1}{n}$ are written on the board. It is allowed to erase any two numbers a and b and replace them with the number $ab + a + b$. What number remains after n–1 such operations?

4. Borgov places white bishops on a chess board, and Beth Harmon places black bishops in her turn, starting with Borgov, in such a way that a new bishop can only be placed on a square that **can** be "taken" by a bishop of the other color already on the board. A player loses if they cannot place a bishop during their turn. Give a strategy for Beth Harmon to win.

5. Four grasshoppers are sitting at the vertices of a square. Every minute one of them jumps to a point symmetrical to it relative to some other grasshopper. Can the grasshoppers at some point end up at the vertices of a larger square?

6. Given 11 red chips, 30 white chips and 19 blue chips, we can replace any two chips of two different colours, by two chips of the third colour. (For example, we may replace a white chip and a blue chip by two red chips.) Can we ever have the same number of chips of two different colours?

7. In a 10×10 square table, nine 1×1 cells are shaded. After that, you can sequentially shade the cells that have at least two adjacent cells (i.e., that have a common side) already shaded. Prove that it is impossible to shade all the cells.

8. There are two three-liter vessels. One contains 1 liter of water, the other contains 1 liter of a two-percent solution of table salt. It is allowed to pour any part of the liquid from one vessel to the other, and then mix. Is it possible to obtain a 1.5% solution in the vessel that initially contained water after several such pourings?